Scale Invariant Feature Transform
: SIFT

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ISL Lab Seminar

Contents

Introduction

illumination

illumination + Scale

illumination + Scale + Rotation

illumination + Scale Rotation + Affine

Introduction

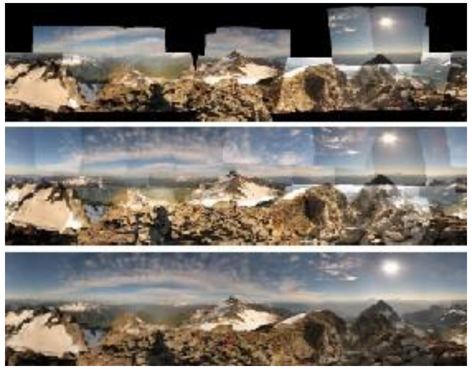
David G Lowe

A senior research scientist at Google (Seattle) in the Machine Intelligence Group.

99: *Object recognition from local scale-invariant features* [1]

04: *Distinctive Image Features from Scale-Invariant Keypoints* [2]

[Overview of Research Projects]



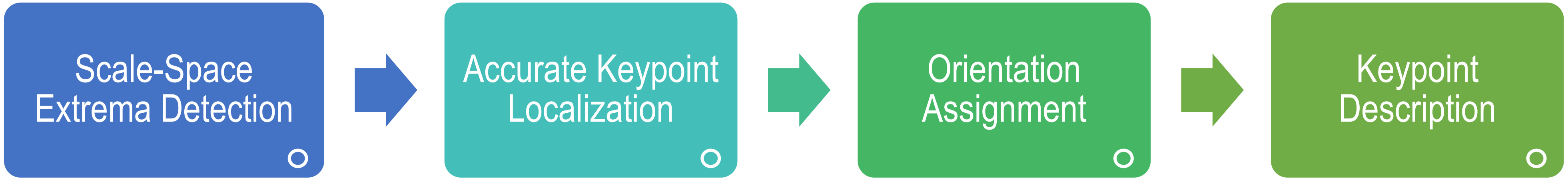
Autostich
: Atuomated paranoma creation

SIFT
: Matching with local invariant features

Augmented reality in natural scenes

[1] *Lowe, David G. "Object recognition from local scale-invariant features." Computer vision, 1999. The proceedings of the seventh IEEE international conference on. Vol. 2. Ieee, 1999.*
[2] *Lowe, David G. "Distinctive image features from scale-invariant keypoints." International journal of computer vision 60.2 (2004): 91-110.*

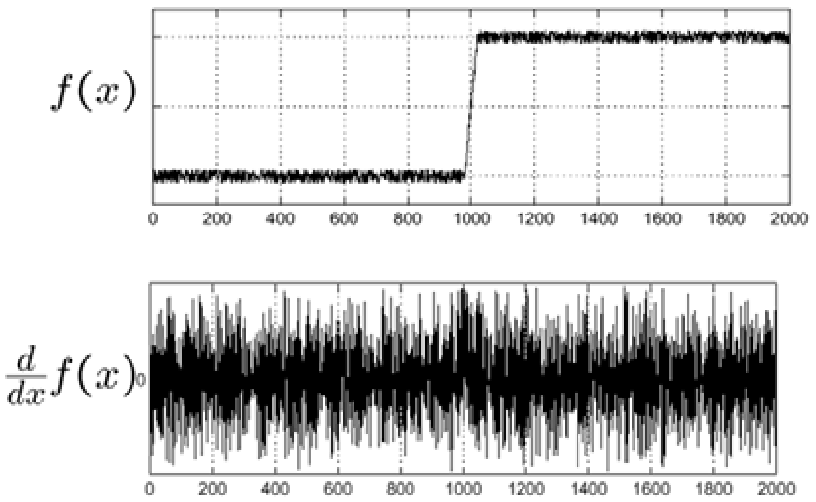
Introduction

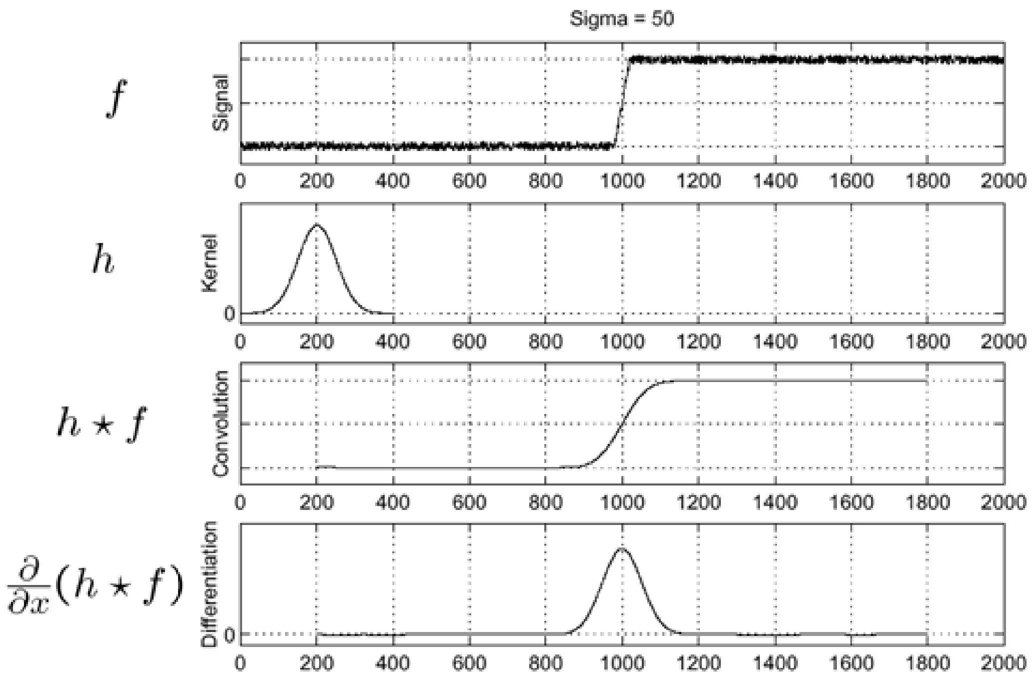


- Search over multiple scales and Image locations.
- Select keypoints based on a measure of stability.
- Compute best orientation(s) for each keypoint region.
- Use local image gradients at selected scale and rotation to describe each keypoint region.

Fundamental theory

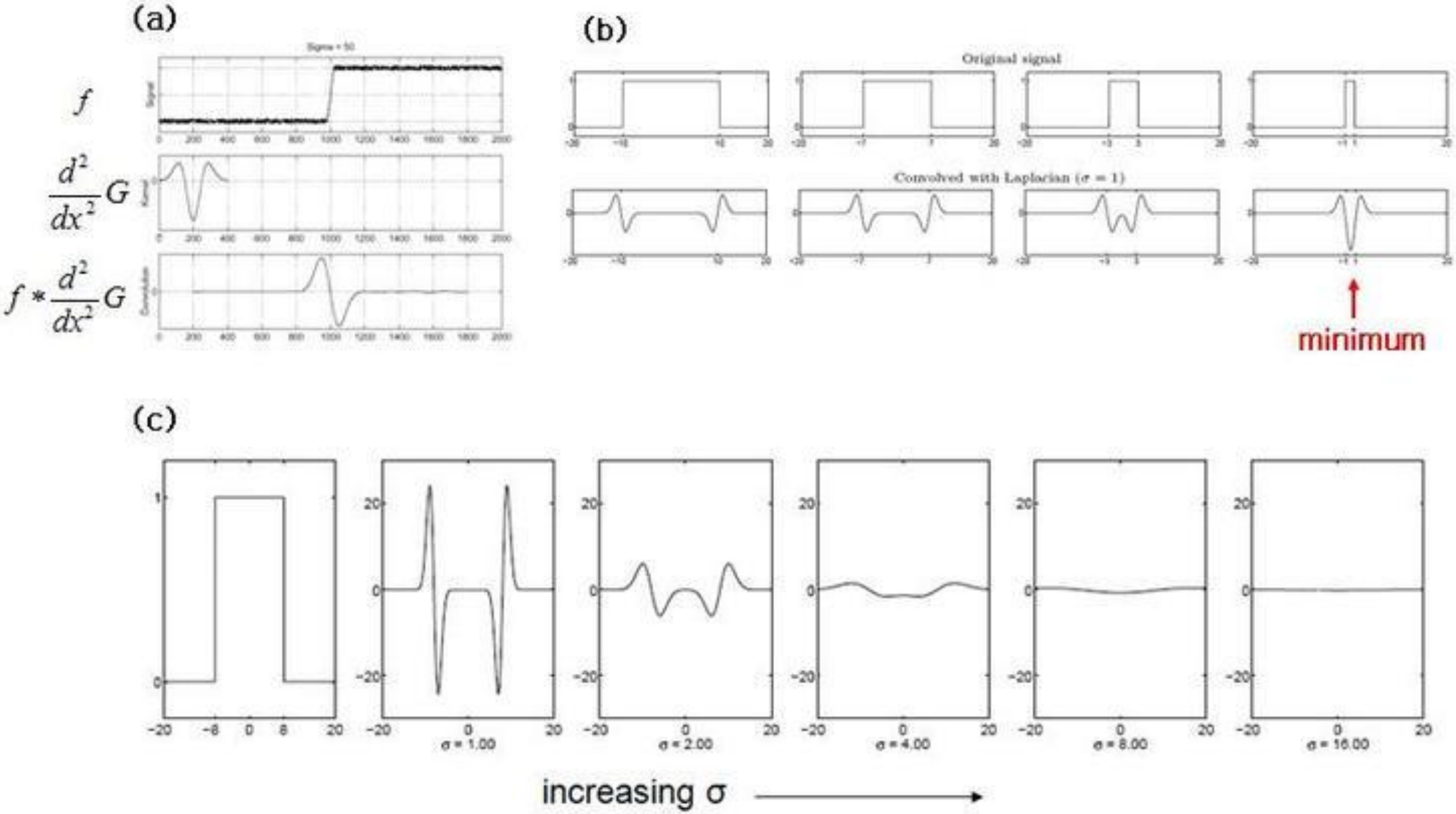
- DOG (edge)





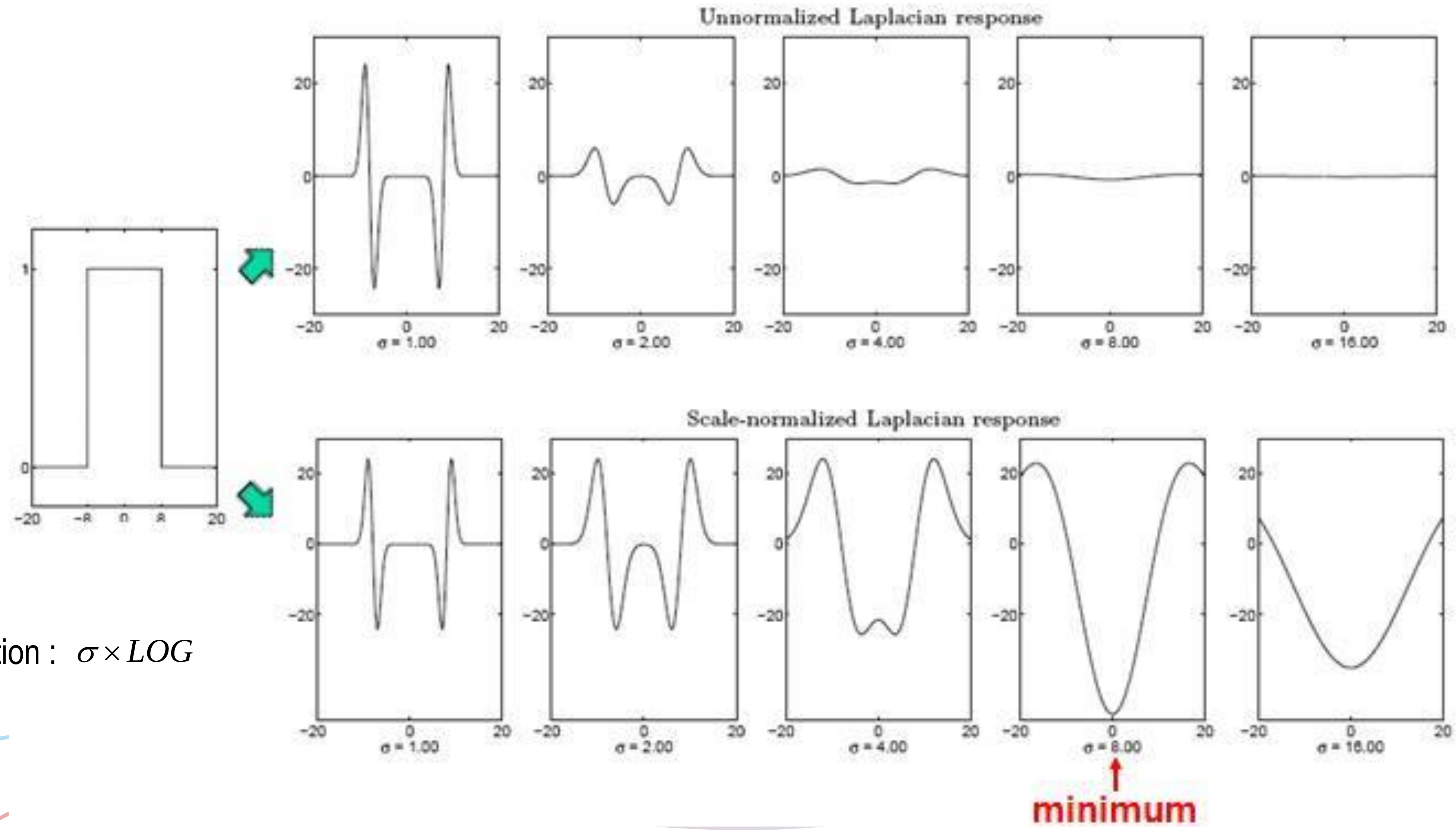
Fundamental theory

❖ LOG (blob)



Fundamental theory

❖ Normalized LOG



Normalization : $\sigma \times LOG$

Fundamental theory

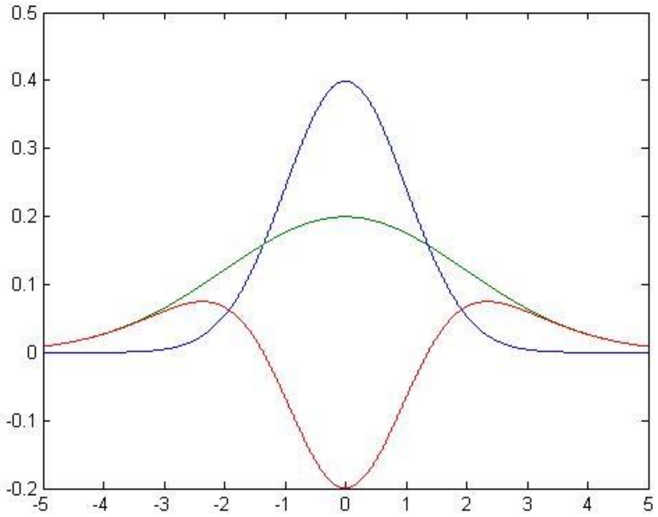
❖ DOG & LOG

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G \longrightarrow \text{Heat Diffusion Equation}$$

$$\frac{\partial G}{\partial \sigma} \approx \frac{G(x,y,k\sigma) - G(x,y,\sigma)}{k\sigma - \sigma}$$

$$G(x,y,k\sigma) - G(x,y,\sigma) \approx (k-1)\sigma^2 \nabla^2 G$$

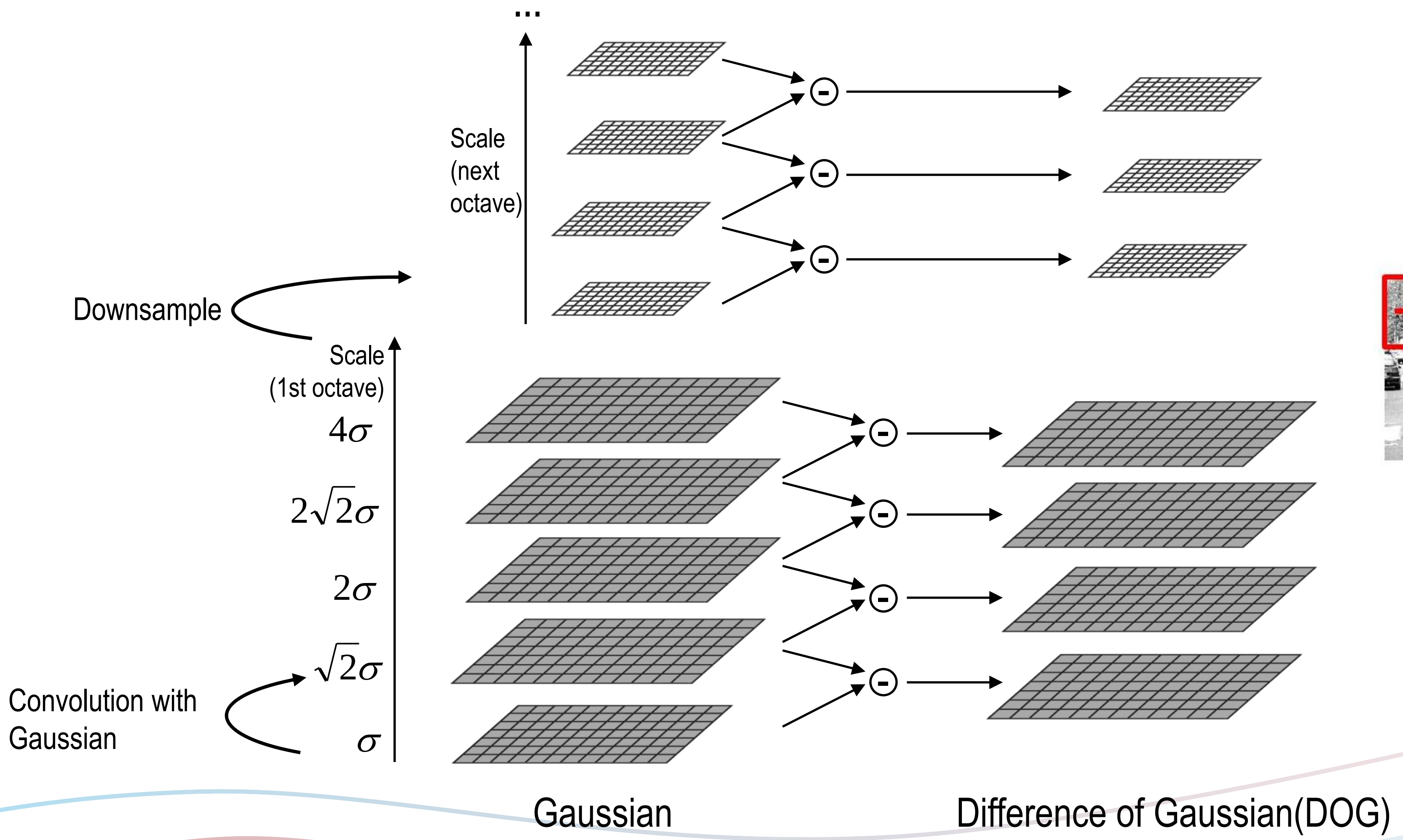
$$DOG \approx (k-1)\sigma^2 \nabla^2 G = (k-1)\sigma \times NLOG(Normalized\ \ Laplacian\ of\ \ Gaussian)$$

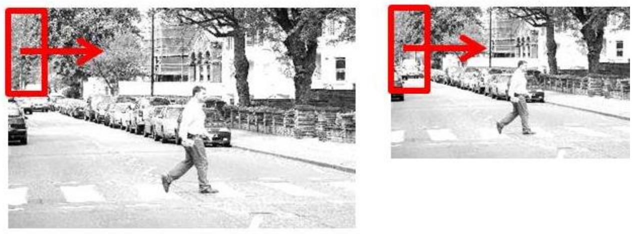


: $\sigma = 1$
: $\sigma = 2$
: DOG

Fundamental theory

❖ Gaussian Pyramid

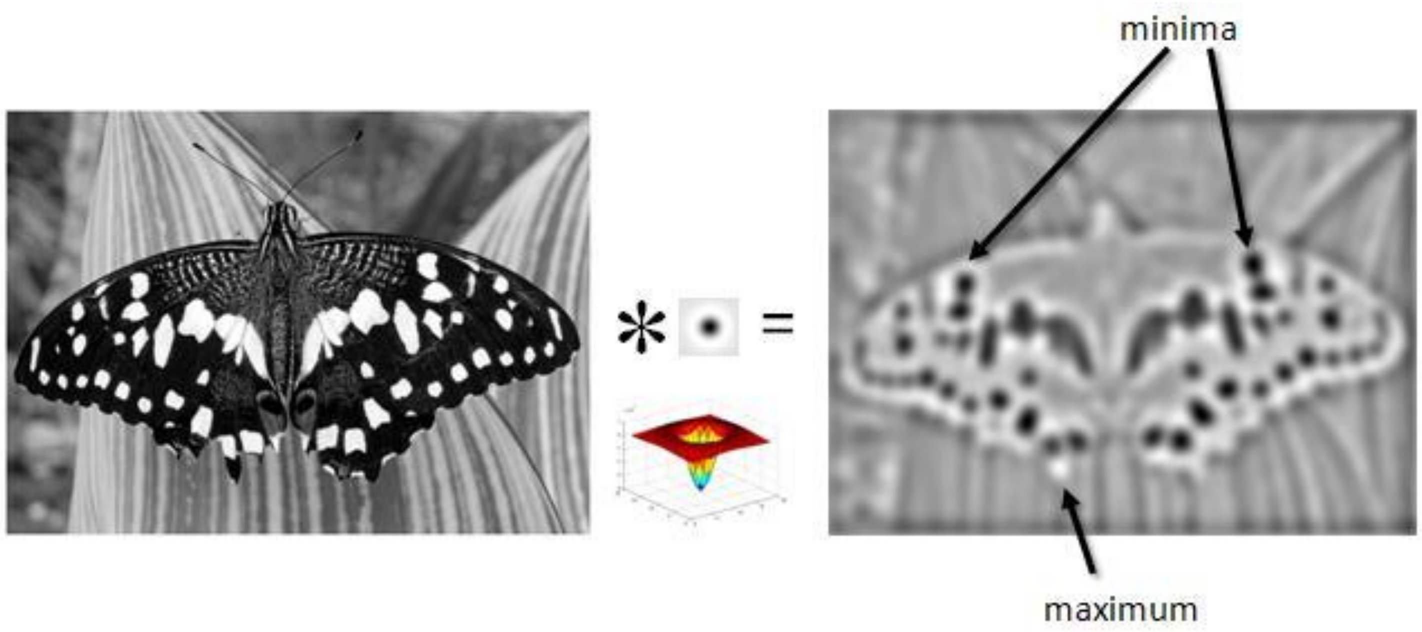




SIFT

❖ Detection of Scale-Space Extrema

Extrema : maxima & minima



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❖ Detection of Scale-Space Extrema

$$L(x,y,\sigma) = G(x,y,\sigma) * I(x,y)$$

$$G(x,y,\sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

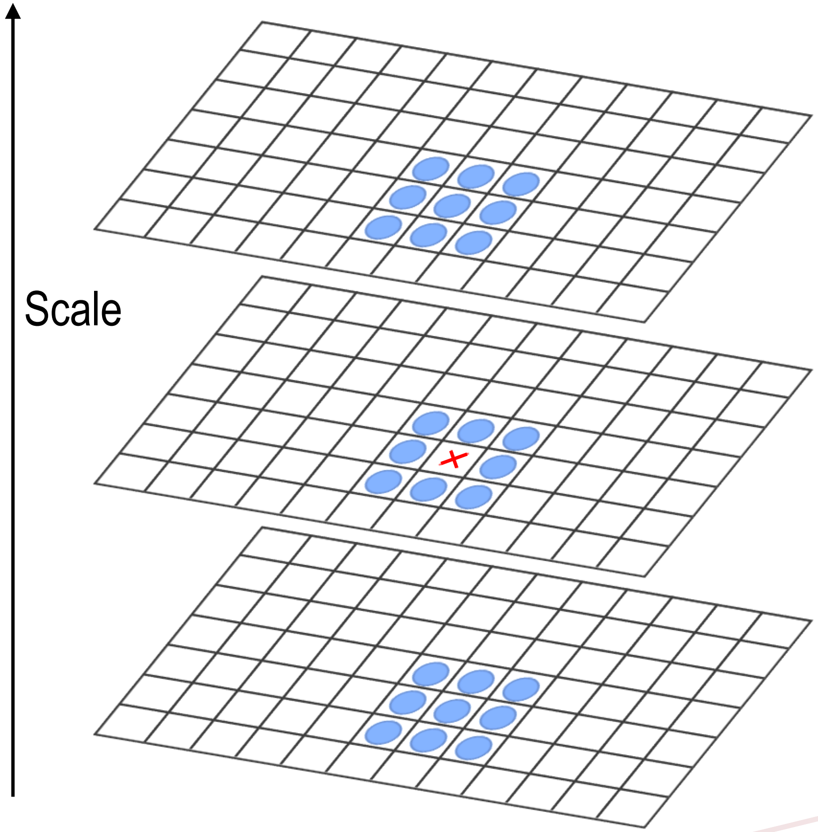
$$\begin{aligned} D(x,y,\sigma) &= (G(x,y,k\sigma) - G(x,y,\sigma)) * I(x,y) \\ &= L(x,y,k\sigma) - L(x,y,\sigma) \end{aligned}$$

$$G(x,y,k\sigma) - G(x,y,\sigma) \approx k(\sigma - 1)\nabla^2 G$$

interval : s

the number of Gaussian Image : $s+3$

scaling ratio : $k = 2^{1/s}$



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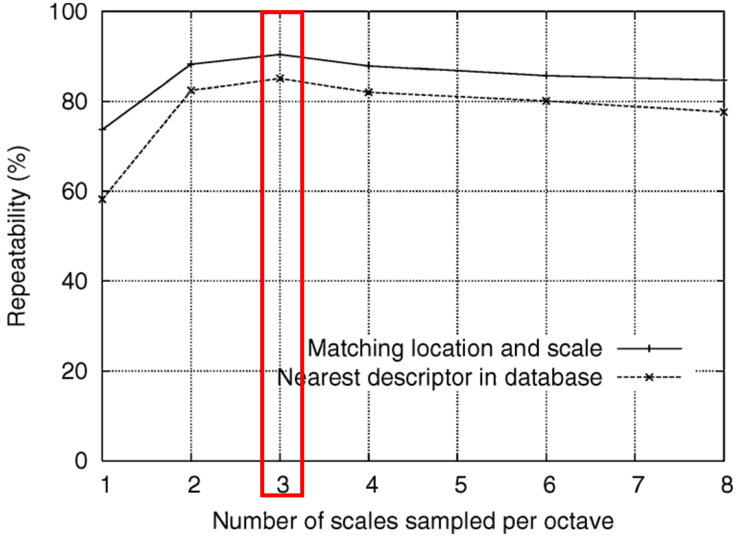
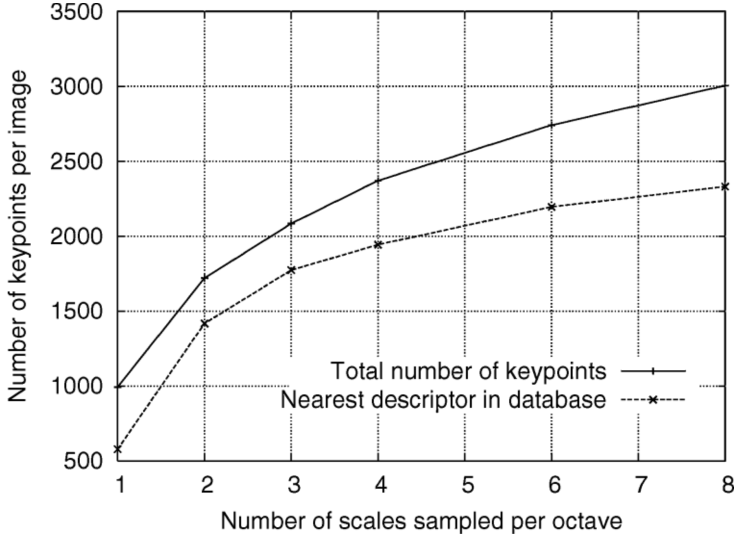
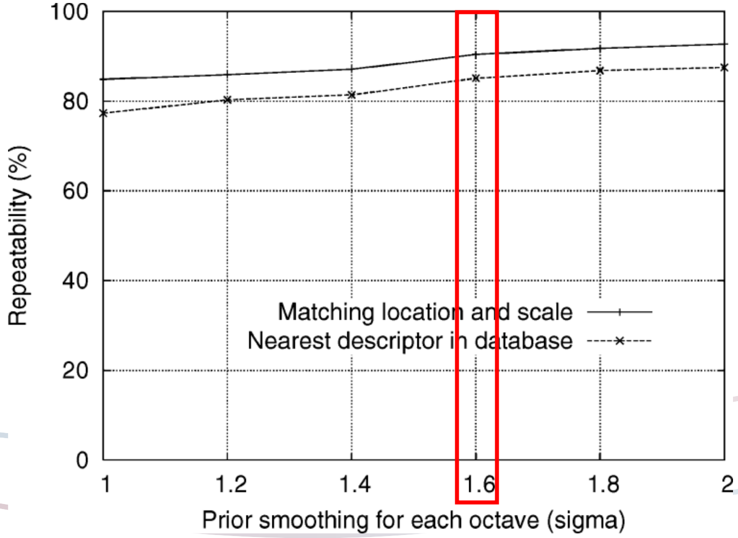
Detection of Scale-Space Extrema







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❖ Accurate Keypoint Localization (low contrast)

$$D(\mathrm{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^{\mathrm{T}}\frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x} \longrightarrow \text{Taylor Expansion}$$

$$D'(\mathrm{x}) = 0 + \frac{\partial D^T}{\partial \mathbf{x}} + \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x} \qquad (\mathbf{x} = (x,y,\sigma)^{\mathrm{T}})$$

$$\frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x} = -\frac{\partial D^T}{\partial \mathbf{x}}$$

$$\mathbf{x} = -\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2}\frac{\partial D^T}{\partial \mathbf{x}} \longrightarrow \hat{\mathbf{x}} = -\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2}\frac{\partial D}{\partial \mathbf{x}} \qquad (if\ \ \hat{\mathbf{x}} > 0.5)$$

$$D(\hat{\mathbf{x}}) = D + \frac{\partial D^T}{\partial \hat{\mathbf{x}}}\hat{\mathbf{x}} + \frac{1}{2}\hat{\mathbf{x}}^T\left(-\frac{\partial D^T}{\partial \hat{\mathbf{x}}}\right)$$

$$= D + \frac{\partial D^T}{\partial \hat{\mathbf{x}}}\hat{\mathbf{x}} - \frac{1}{2}\frac{\partial D^T}{\partial \hat{\mathbf{x}}}\hat{\mathbf{x}}$$

$$= D + \frac{1}{2}\frac{\partial D^T}{\partial \hat{\mathbf{x}}}\hat{\mathbf{x}} \qquad \therefore D(\hat{\mathbf{x}}) = D + \frac{1}{2}\frac{\partial D^T}{\partial \mathbf{x}}\hat{\mathbf{x}} \qquad (if\ \left|D(\hat{\mathbf{x}})\right| < 0.03)$$

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❖ Accurate Keypoint Localization (edge)

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \longrightarrow$$ Hessian Matrix

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta \qquad \mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta \qquad (\alpha > \beta)$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r} \qquad (r = 10)$$

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❖ Accurate Keypoint Localization

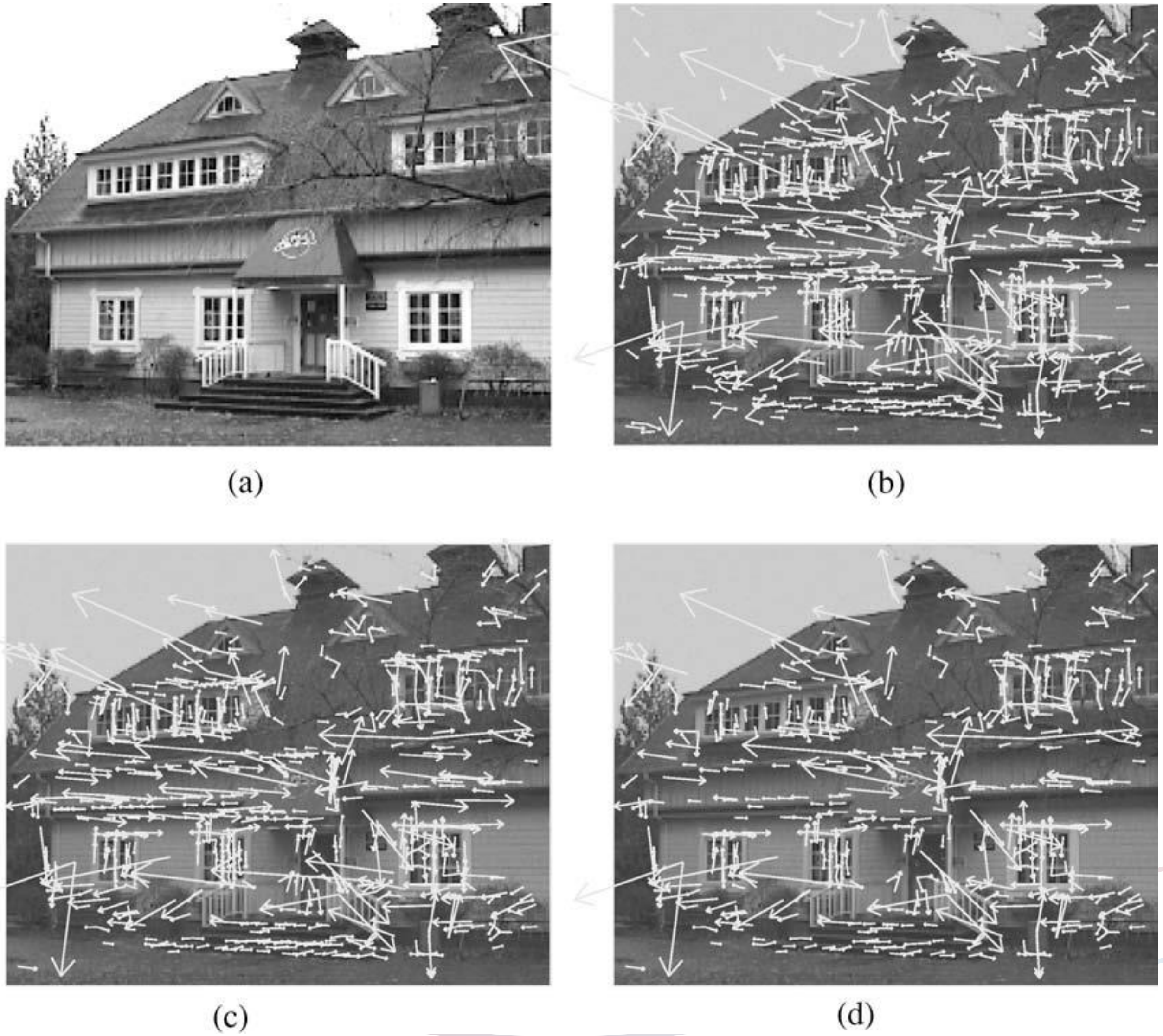
(a) 233x189 pixel original image

(b) 832 keypoints location

(c) 729 keypoints location
(threshold on minimum contrast)

(d) 536 keypoints location

(threshold on ratio of principal curvatures)



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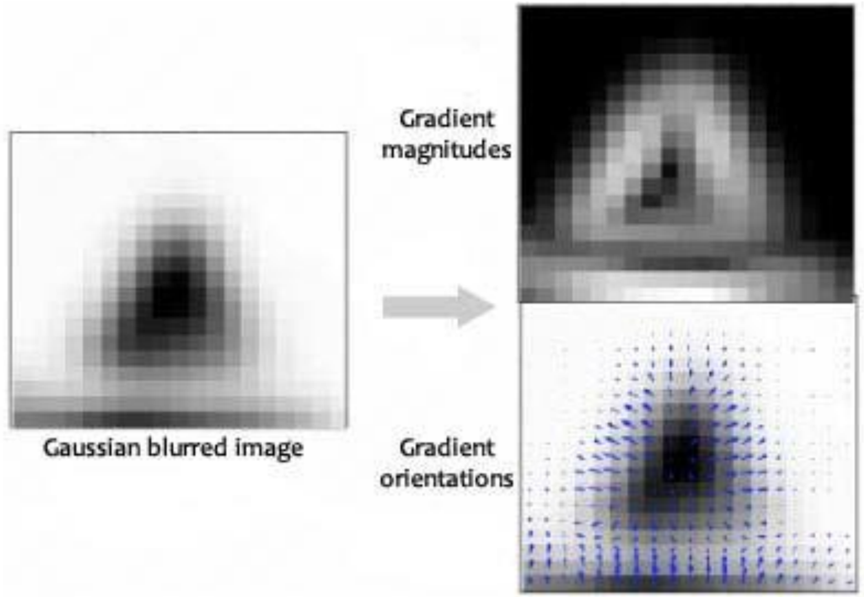
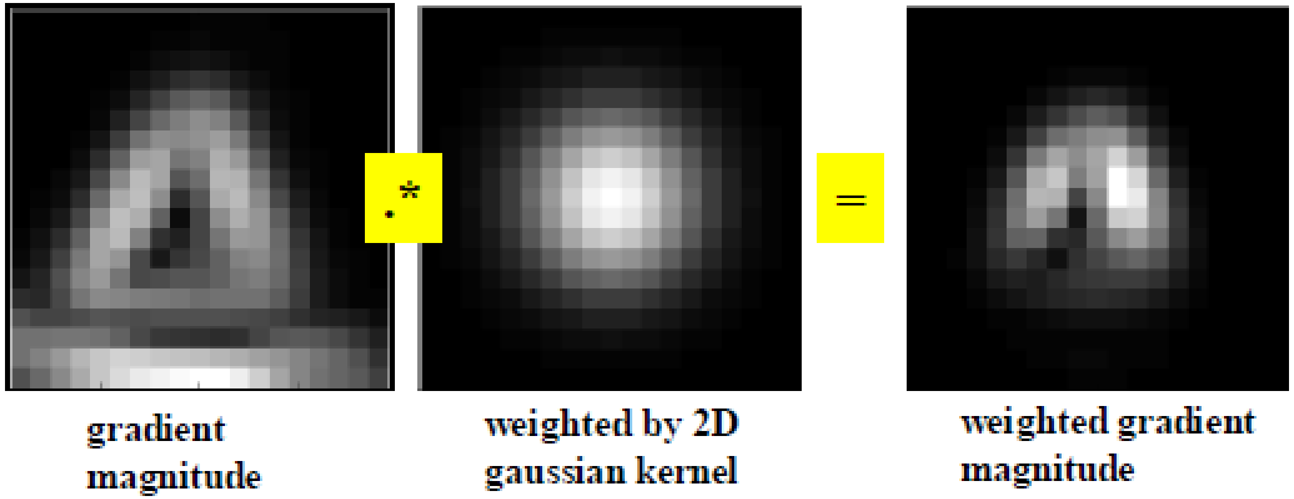
❖ Orientation Assignment

$$m(x,y)=\sqrt{(L(x+1,y)-L(x-1,y))^2+(L(x,y+1)-L(x,y-1))^2}$$

$$\theta(x,y)=\tan^{-1}((L(x,y+1)-L(x,y-1))/(L(x+1,y)-L(x-1,y)))$$





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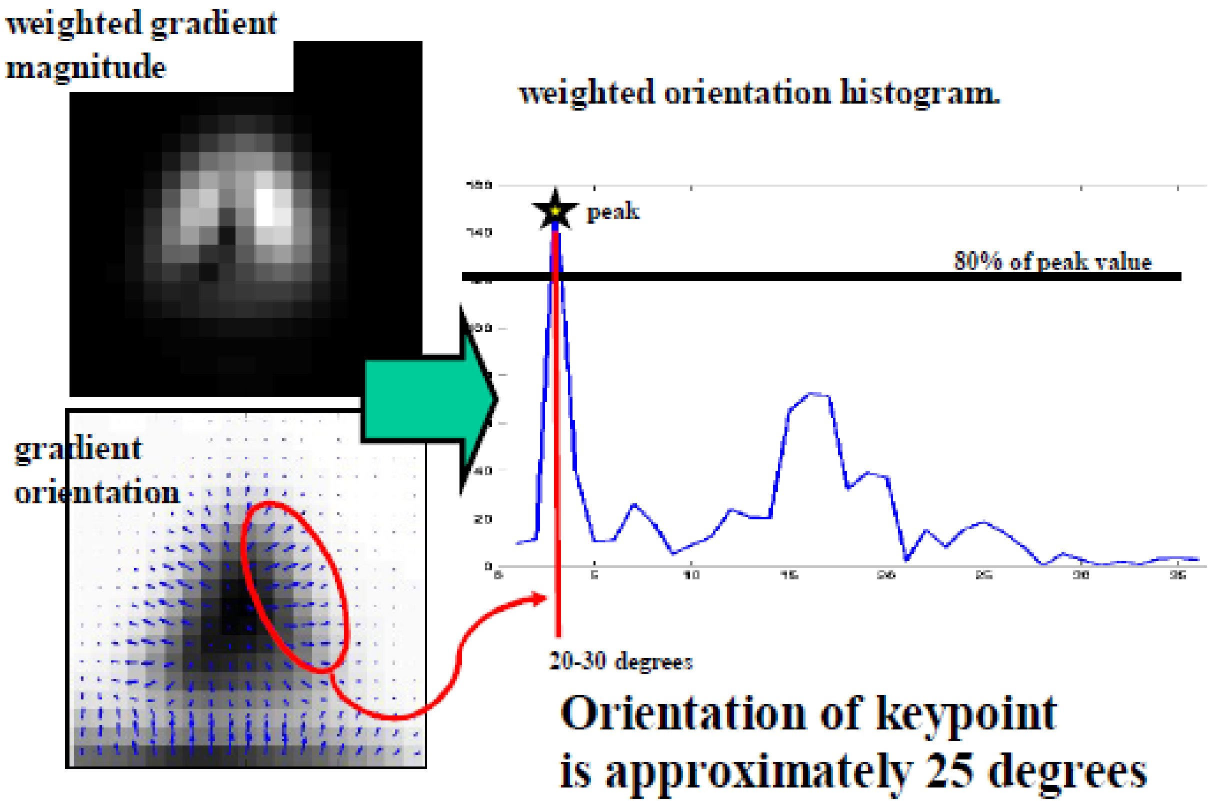
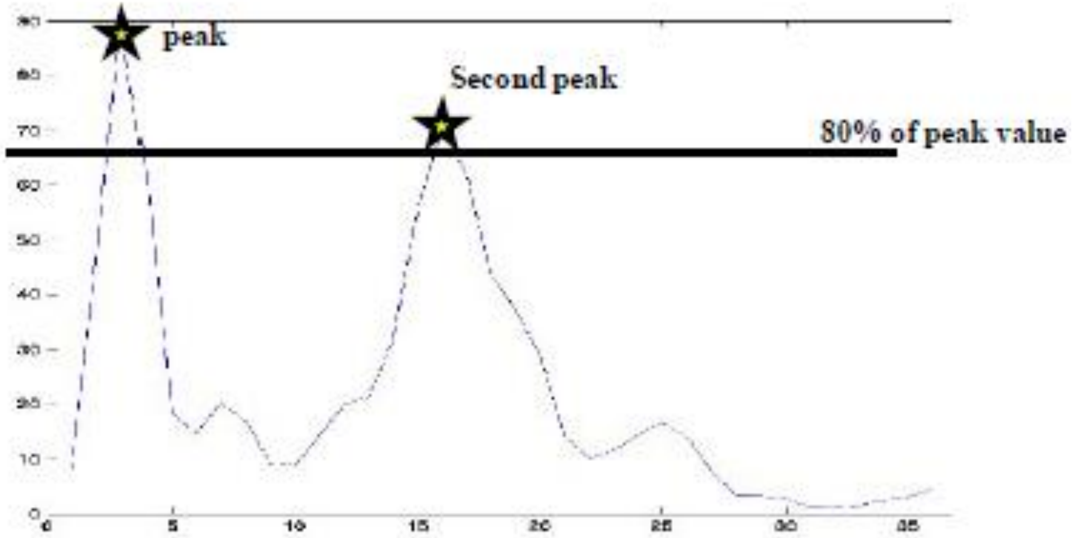
❖ Orientation Assignment





Histogram : Using 36bins

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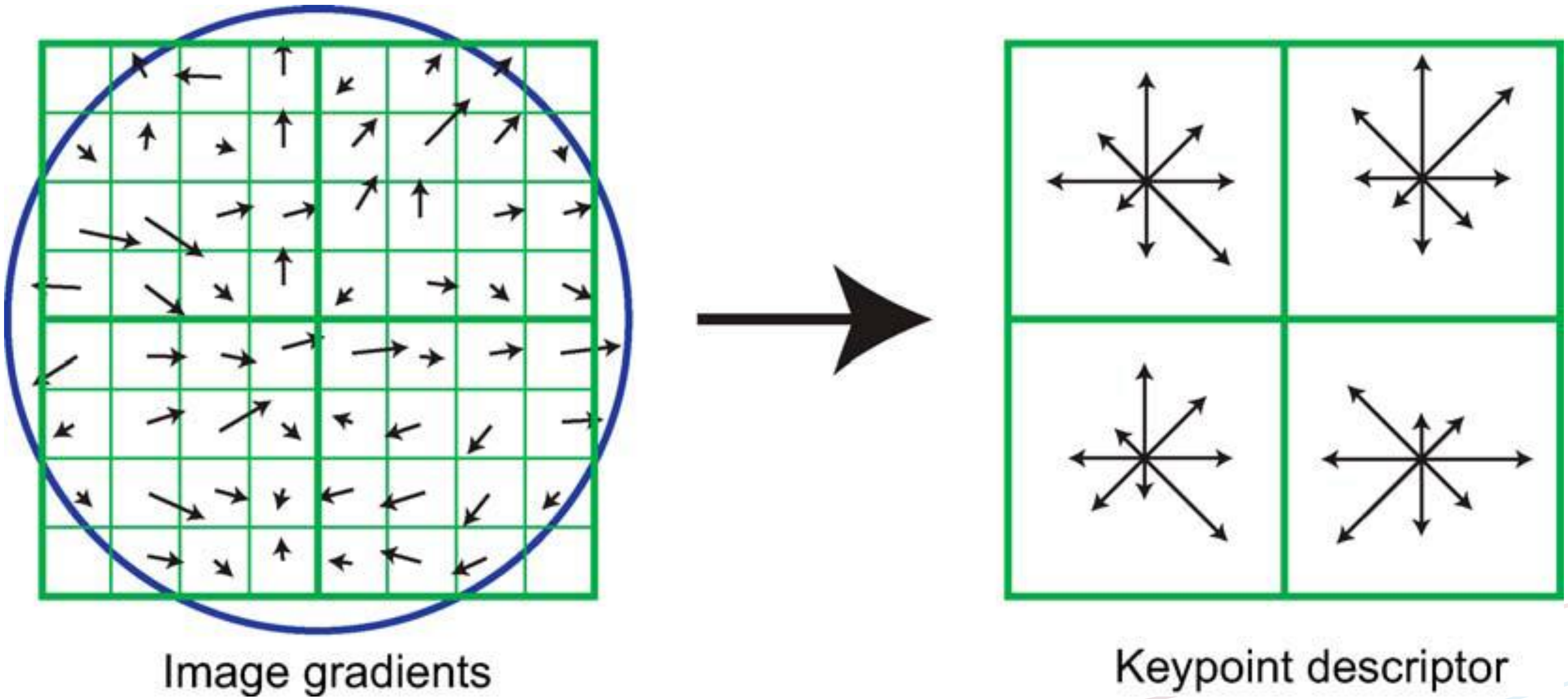
❖ The Local Images Descriptor



illumination : normalization vector
(Feature vector < 0.2)

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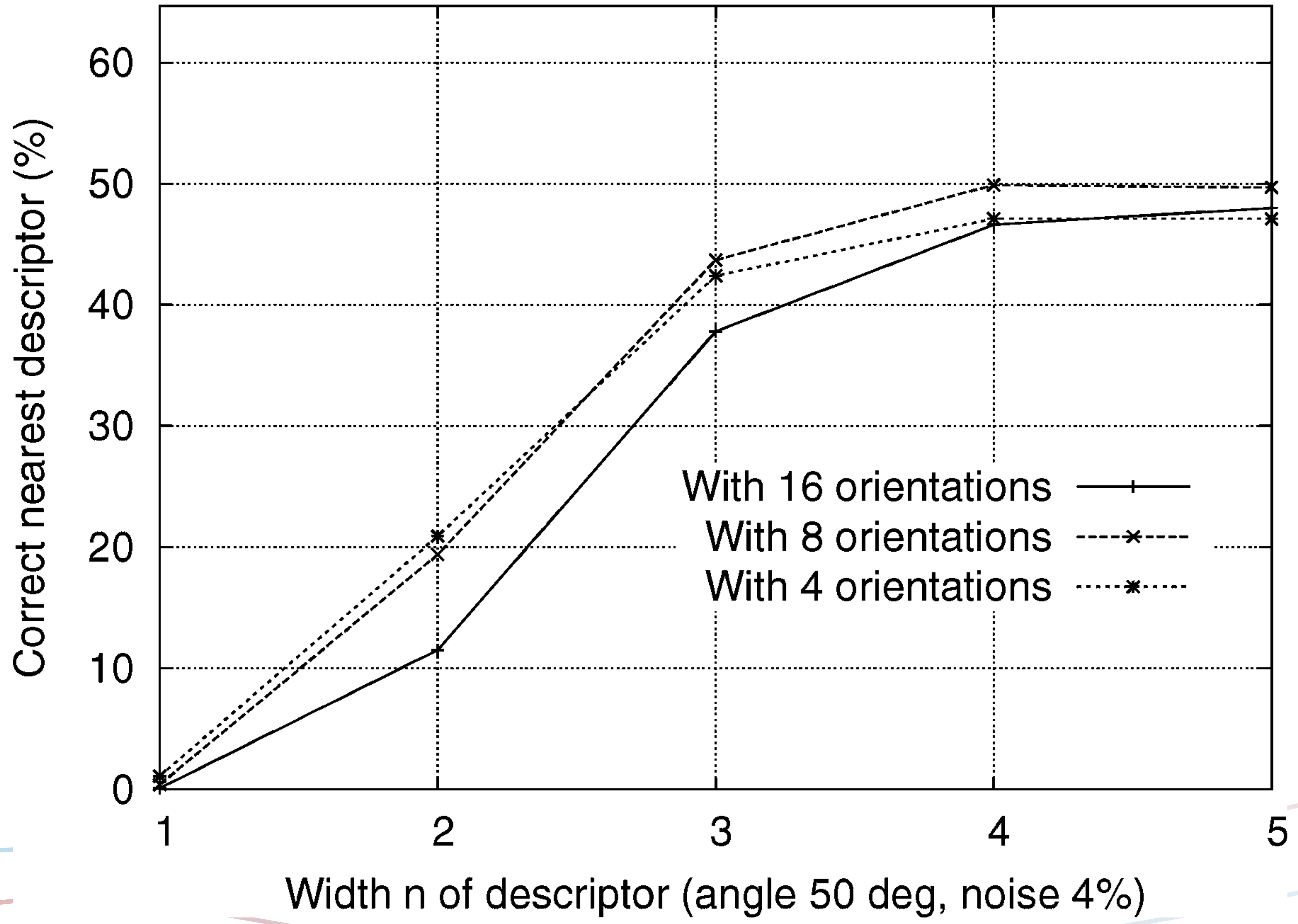
❖ **The Local Images Descriptor**



r : the number of orientations
n : the width

The size of the resulting descriptor vector is rn^2

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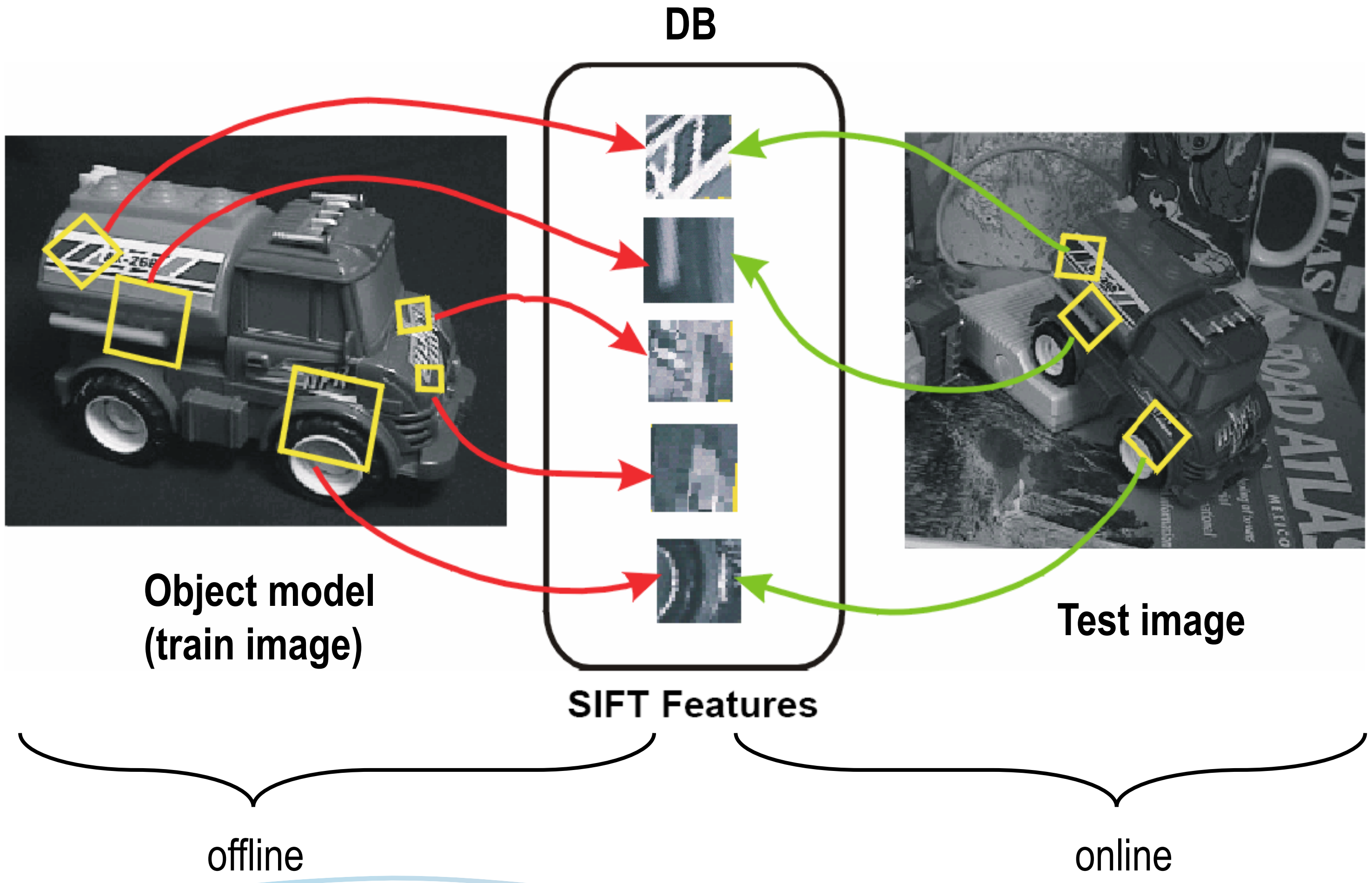
❖ Keypoint Matching



1) Nearest-neighbor search
   : Euclidean distance, K-D tree, BBF(Best-Bin-First)
2) Cluster identification by Hough transform voting
3) Model verification by linear least squares
4) Outlier detection

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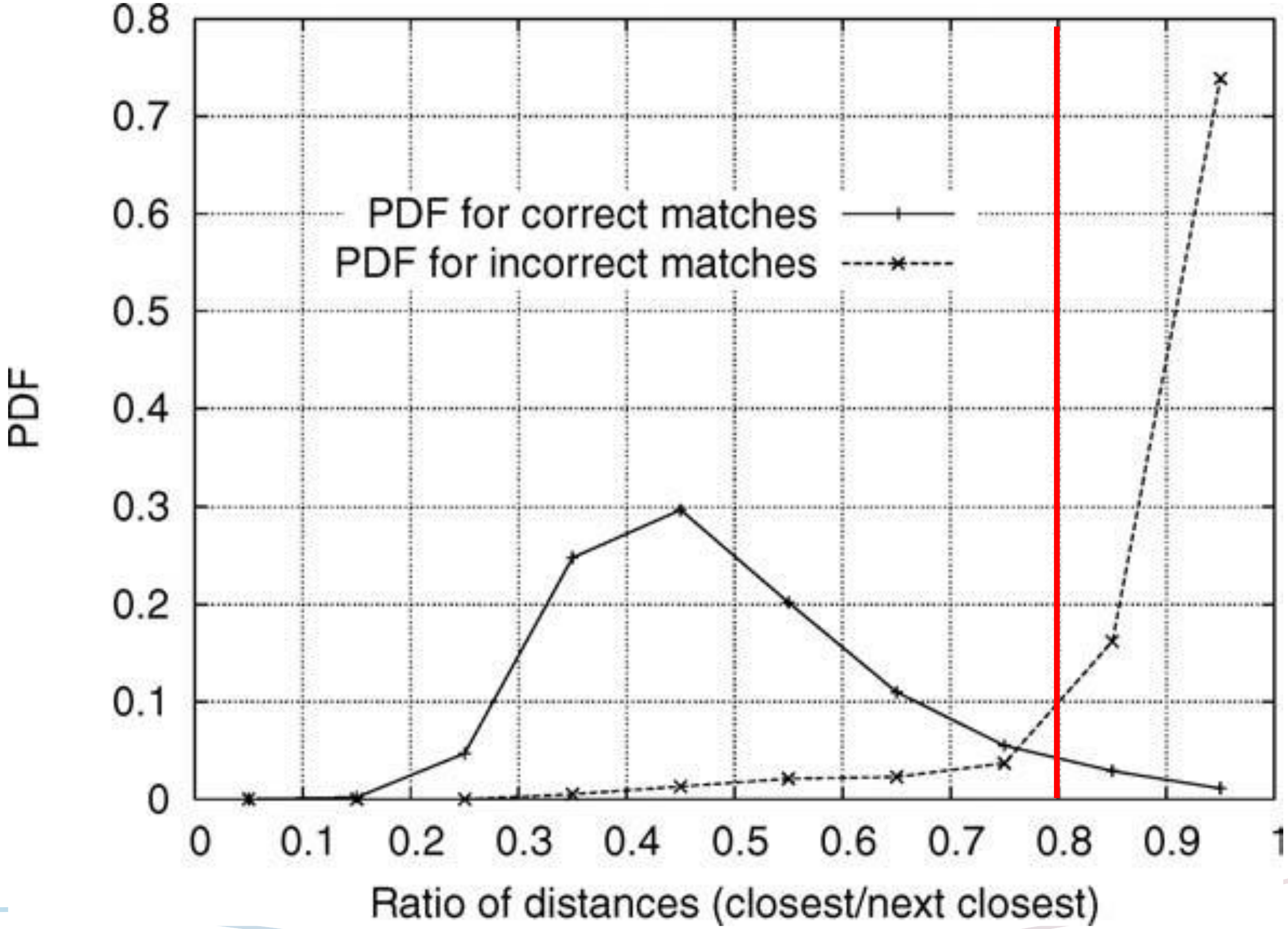
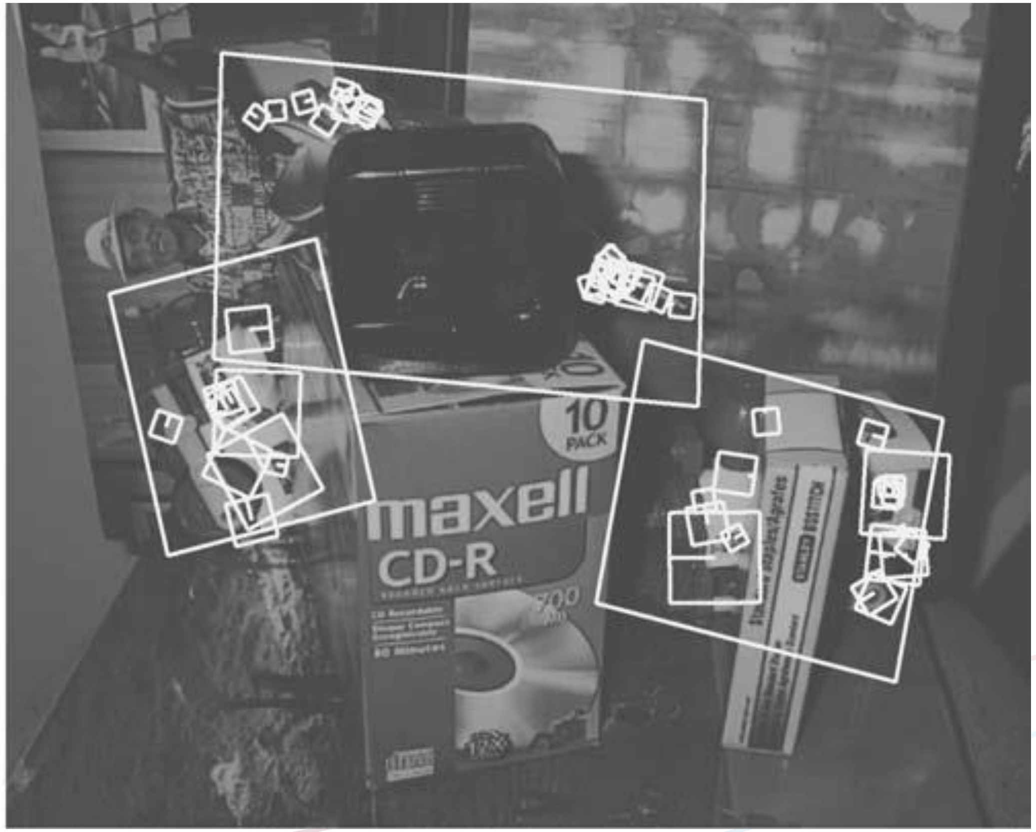
❖ Keypoint Matching



Example

- Recognition



Example

- Recognition

Q & A

Thank You!!

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} m1 & m2 \\ m3 & m4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} x & y & 0 & 0 & 1 & 0 \\ 0 & 0 & x & y & 0 & 1 \\ & & ... & & & \\ & & ... & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix} = \begin{bmatrix} u \\ v \\ \vdots \end{bmatrix}$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = [\mathbf{A}^{\mathrm{T}}\mathbf{A}]^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$