Poisson Matting

※ Sun, Jian, et al. "Poisson matting." *ACM Transactions on Graphics (ToG)* 23.3 (2004): 315-321.

ISL
안재원

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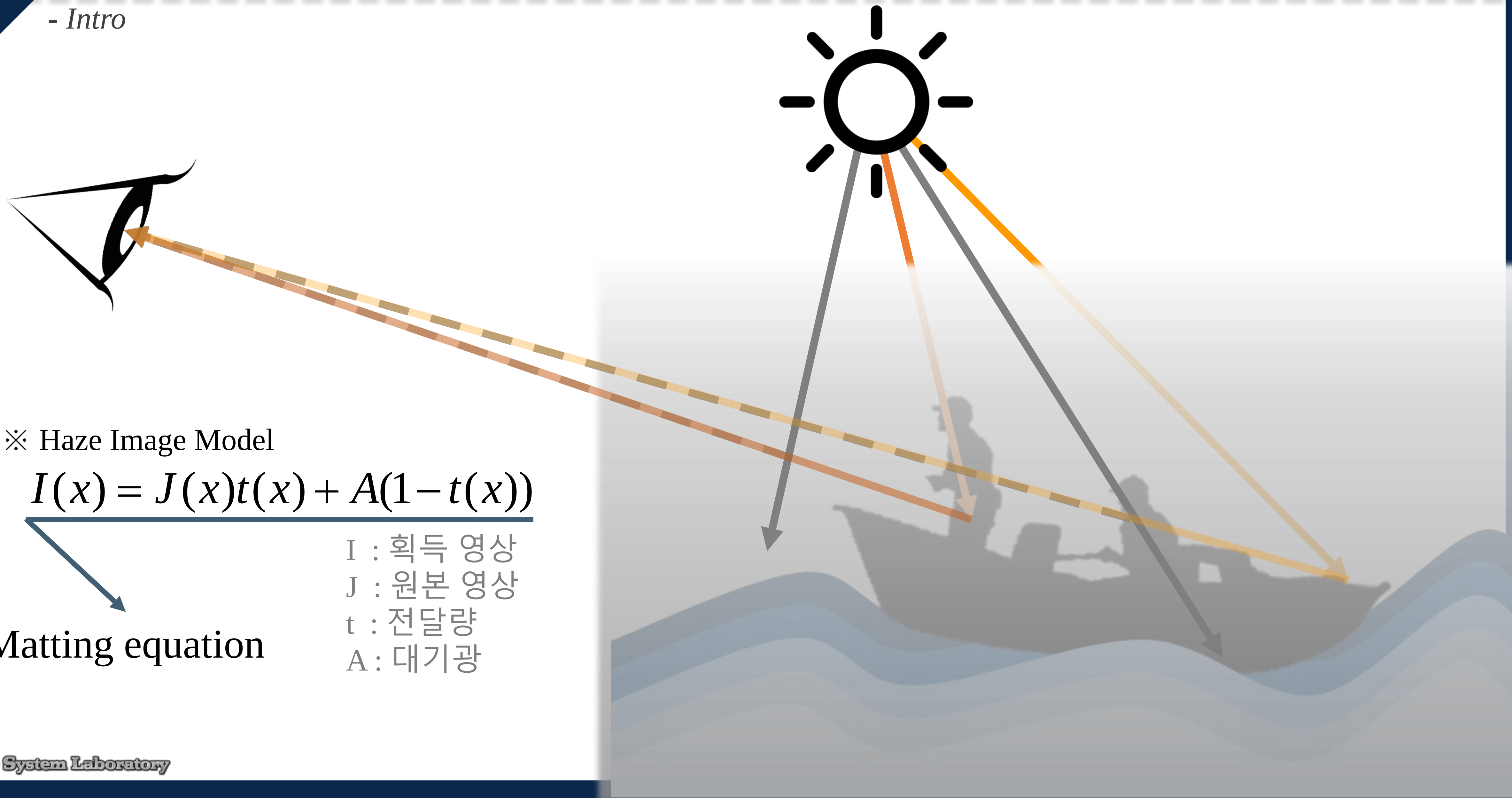
- Matting
- Poisson Equation
- Poisson Matting
- Result

Matting

- *Intro*



※ Haze Image Model

$$I(x) = J(x)t(x) + A(1 - t(x))$$

Matting equation

I : 획득 영상
J : 원본 영상
t : 전달량
A : 대기광

Matting

- *Matting example*

Matting

- *Matting equation*

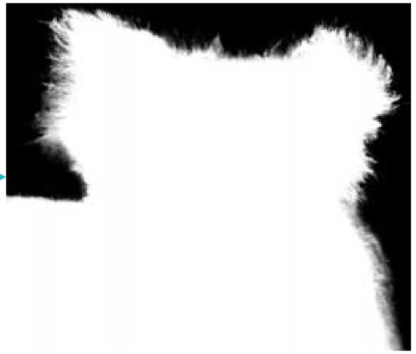
※ Haze Image Model

$$I(x) = J(x)t(x) + A(1 - t(x))$$

※ Matting equation

$$I = \alpha F + (1 - \alpha)B$$

I : 획득 영상
F : Foreground
B : BackGround
α : Alpha channel



Matting

- *Matting*

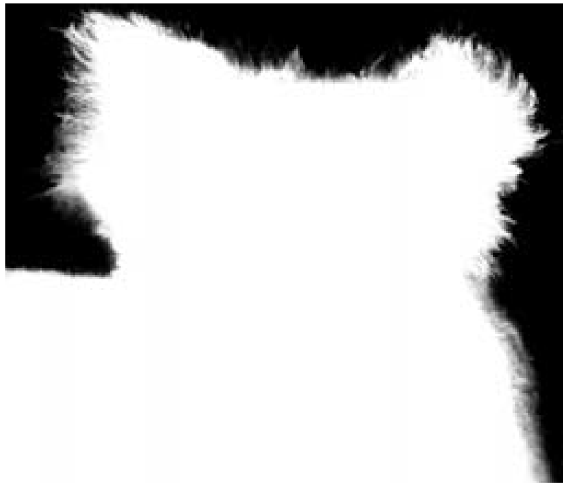
Input Image

Background

User Trimap

Foreground

Unknown
- Probabilistic Model
- Bayesian Model
- Poisson Model
- Closed Form Model



Alpha channel

01

Matting

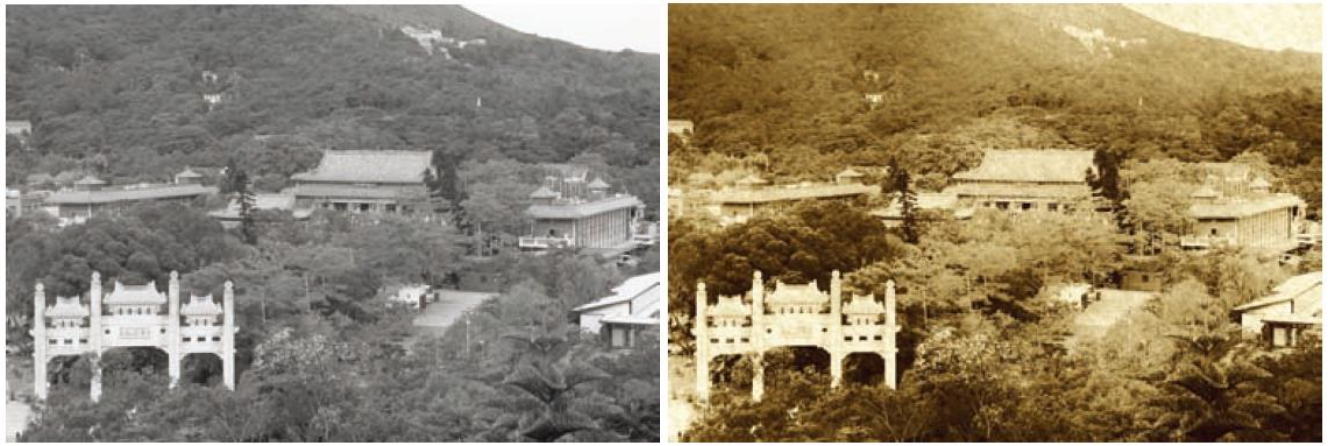
- Problem

Poisson Equation

- *Usage*

<The Poisson Equation>

1. Shadow removal
2. Tone mapping
3. Image editing
4. Surface reconstruction



Poisson Equation

- *Equation*

- The Poisson Equation

Second order PDE(Partial Differential Equation)

$$\Delta\varphi = \nabla^2\varphi = f$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi(x,y) = f(x,y)$$

- Estimate Alpha channel

$$\Delta\alpha = div\left(\frac{\nabla I}{(F-B)}\right)$$

- Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \cdots, \frac{\partial f}{\partial x_n}\right)^T$$

- Divergence

$$\vec{\nabla} \cdot \vec{f} = \frac{\partial \vec{f}}{\partial x_1} + \cdots + \frac{\partial \vec{f}}{\partial x_n}$$

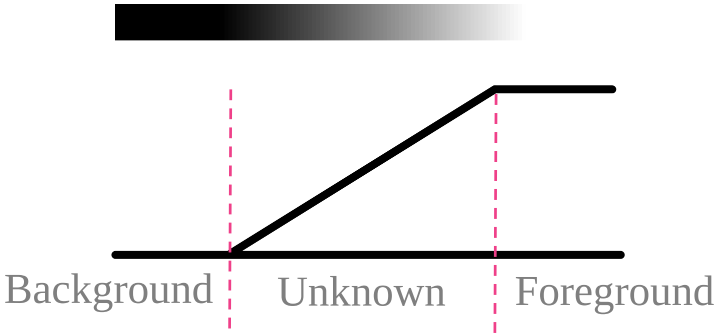
Poisson Matting

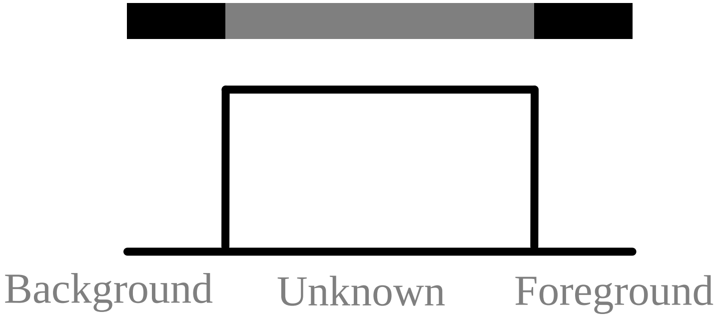
- *Intro*

<Assumption>

- Locally smooth(foreground, background)

$$\begin{cases} \nabla F \approx 0 \\ \nabla B \approx 0 \end{cases}$$





<Overview>

1. Start with a user trimap
2. Estimate alpha values in unknown area.
3. Refine trimap
4. Back to '2.'

Poisson Matting

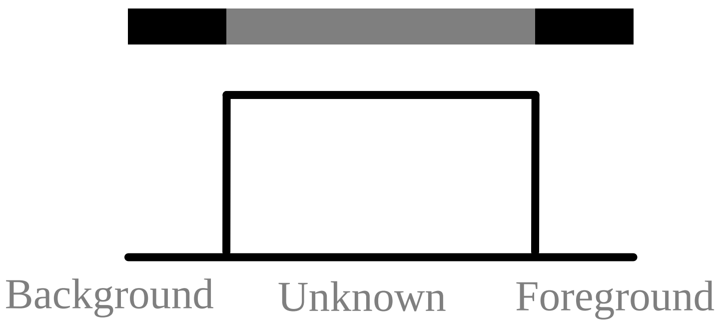
- *Calculating α*

※ Matting equation

$$I = \alpha F + (1-\alpha)B$$

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

$$\nabla I = (F-B)\nabla\alpha$$



$$\therefore \nabla\alpha \approx \frac{\nabla I}{(F-B)}$$

1. Alpha channel 과 image의 gradient는 비례함.
2. 적분을 통해 Alpha channel을 구할 수 있음.

Poisson Matting

- *Calculating α*

※ Pérez, Patrick, Michel Gangnet, and Andrew Blake. "Poisson image editing."*ACM Transactions on Graphics (TOG)*. Vol. 22. No. 3. ACM, 2003.

- Variational problem(Guided interpolation)

$$\alpha^* = \arg\min_{\alpha} \iint_{p\in\Omega} \left\| \nabla \alpha_p - \frac{\nabla I_p}{(F_p - B_p)} \right\|^2 dp$$

F_p : Nearest foreground pixel

B_p : Nearest background pixel

- Poisson Equation

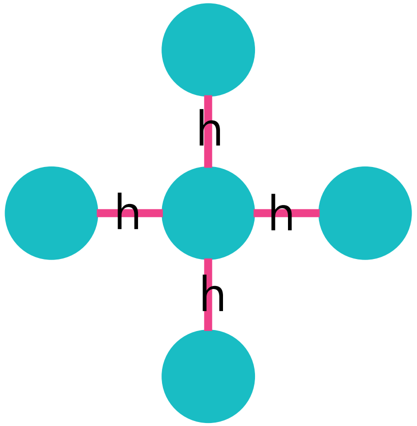
$$\Delta\alpha = div\left(\frac{\nabla I}{F - B}\right)$$

Poisson Matting

- *Calculating α*

- Poisson Equation

$$\Delta\alpha = div\left(\frac{\nabla I}{F - B}\right) \;\Rightarrow\; \frac{\partial^2\alpha}{\partial x^2} + \frac{\partial^2\alpha}{\partial y^2} = f(x,y)$$

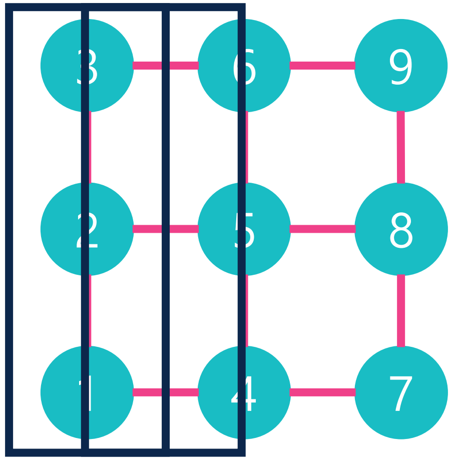


$$\frac{\alpha(x+h,y)+\alpha(x-h,y)+\alpha(x,y+h)+\alpha(x,y-h)-4\alpha(x,y)}{h^2} = f(x,y)$$

$$\begin{bmatrix}1 & 1 & -4 & 1 & 1\end{bmatrix}\begin{bmatrix}\alpha(x+h,y)\\ \alpha(x-h,y)\\ \alpha(x,y)\\ \alpha(x,y-h)\\ \alpha(x,y+h)\end{bmatrix} = f(x,y)$$

Poisson Matting

- *Calculating α*



$$
\begin{bmatrix} 1 & 1 & -4 & 1 & 1 \end{bmatrix}
\begin{bmatrix} \alpha(x+h,y) \\ \alpha(x-h,y) \\ \alpha(x,y) \\ \alpha(x,y-h) \\ \alpha(x,y+h) \end{bmatrix}
$$

$$
\left[\begin{array}{ccc|ccc|ccc}
-4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4
\end{array}\right]
\begin{bmatrix} \alpha(x_1,y_1) \\ \alpha(x_2,y_2) \\ \alpha(x_3,y_3) \\ \alpha(x_4,y_4) \\ \alpha(x_5,y_5) \\ \alpha(x_6,y_6) \\ \alpha(x_7,y_7) \\ \alpha(x_8,y_8) \\ \alpha(x_9,y_9) \end{bmatrix}
=
\begin{bmatrix} f(x_1,y_1) \\ f(x_2,y_2) \\ f(x_3,y_3) \\ f(x_4,y_4) \\ f(x_5,y_5) \\ f(x_6,y_6) \\ f(x_7,y_7) \\ f(x_8,y_8) \\ f(x_9,y_9) \end{bmatrix}
$$

→ 연립방정식을 풀어 Unknown영역의 Alpha channel값을 구한다.

Poisson Matting

- *Optimization*

- Refinement

$$\Omega_F^+ = \{p \, in \, \Omega | \alpha_p > 0.95, I_p \approx F_p\}$$
$$\Omega_B^+ = \{p \, in \, \Omega | \alpha_p < 0.05, I_p \approx B_p\}$$

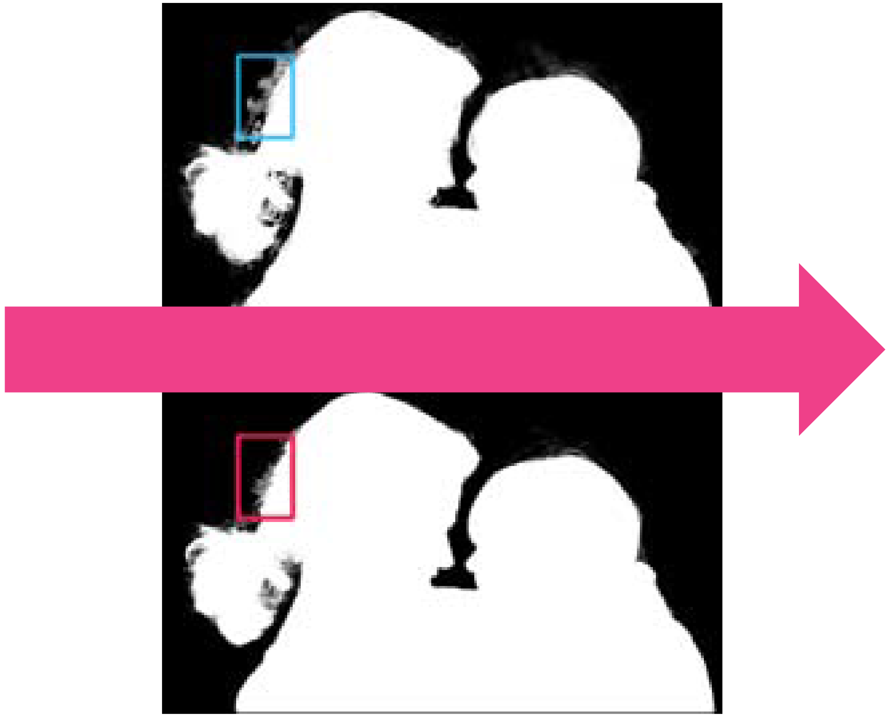
- New trimap

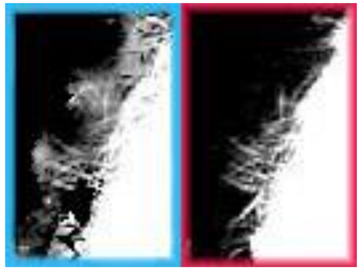
$$\Omega_F = \Omega_F \cup \Omega_F^+$$
$$\Omega_B = \Omega_B \cup \Omega_B^+$$

- 2~3회 반복.

Result

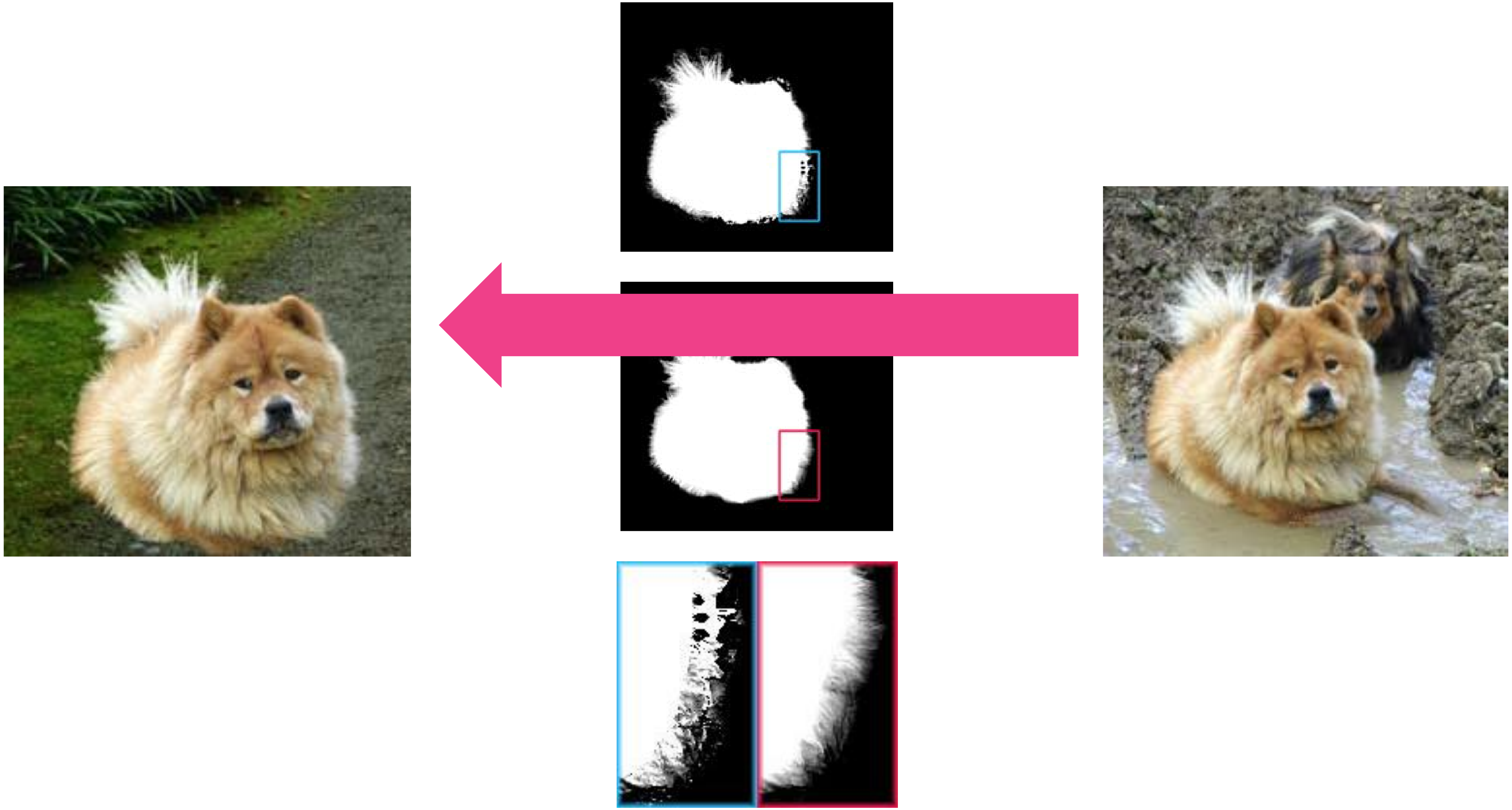
- Intro





Result

- *Intro*



Q & A